THE ORDINARY THEORY OF THE BEAM

For the discussion of the relation of the internal stresses to the external load, assume the simple case of a beam of uniform rectangular cross-section, supported at each end, and loaded at the centre. The lower surface of the beam will become convex, and the upper concave, and the fibres at the lower side are extended, and at the upper side compressed. The surface which separates these two portions tions of the beam and which is neither extended nor compressed is called the *neutral surface.*

Let l = length of span.

h = depth of beam. .

b = breadth of beam.

E = Young's modulus of elasticity.

W = load at centre of beam.

Δ = deflection at centre of beam.

f = stress on extreme fibre.

Then, according to the theory of the beam in common use:—

$$f = \frac{3Wl}{2bh^2} \tag{1}$$

$$\Delta = \frac{Wl^3}{4Ebh^3} = \frac{l^2 f}{6Eh} \tag{2}$$

This theory is founded upon the ordinary laws of statics, and the following hypotheses:—

1. A cross-section, which is plane before bending, remains plane after bending; the form and area of the cross-section is unaltered; the curvature of the beam is continuous.

2. The stresses are connected with the strains by Hooke's Law.

From the first of these hyphotheses is deduced the law that the strain of the fibre is proportional to the distance from the neutral axis. From the second that the stress is also proportional to the distance from the neutral axis, and that the neutral axis is at the centre of the beam (of rectangular cross-section).

They may be accepted as true, provided:—

(*a*) The beam is a "long" beam; i.e., the ratio of length to depth is large so that the shearing stresses may be neglected in comparison with the normal stresses.