

29. If a circle be described about the triangle  $ABC$ , and a straight line be drawn bisecting the angle  $BAC$  and cutting the circle in  $D$ , shew that the angle  $DCB$  will be equal to half the angle  $BAC$ .

30. If the line  $AD$  bisect the angle  $A$  in the triangle  $ABC$ , and  $BD$  be drawn without the triangle making an angle with  $BC$  equal to half the angle  $BAC$ , shew that a circle may be described about  $ABCD$ .

31. Two equal circles intersect in  $A, B$ :  $PQT$  perpendicular to  $AB$  meets it in  $T$  and the circles in  $P, Q$ .  $AP, BQ$  meet in  $R$ ;  $AQ, BP$  in  $S$ ; prove that the angle  $RTS$  is bisected by  $TF$ .

32. If the angle, contained by any side of a quadrilateral and the adjacent side produced, be equal to the opposite angle of the quadrilateral, prove that any side of the quadrilateral will subtend equal angles at the opposite angles of the quadrilateral.

33. If  $DE$  be drawn parallel to the base  $BC$  of a triangle  $ABC$ , prove that the circles described about the triangles  $ABC$  and  $ADE$  have a common tangent at  $A$ .

34. Describe a square equal to the difference of two given squares.

35. If tangents be drawn to a circle from any point without it, and a third line be drawn between the point and the centre of the circle, touching the circle, the perimeter of the triangle formed by the three tangents will be the same for all positions of the third point of contact.

36. If on the sides of any triangle as chords, circles be described, of which the segments external to the triangle contain angles respectively equal to the angles of a given triangle, those circles will intersect in a point.

37. Prove that if  $ABC$  be a triangle inscribed in a circle, such that  $BA=BC$ , and  $AA'$  be drawn parallel to  $BC$ , meeting the circle again in  $A'$ , and  $A'B$  be joined cutting  $AC$  in  $E$ ,  $BA$  touches the circle described about the triangle  $AEA'$ .

38. Describe a circle, cutting the sides of a given square, so that its circumference may be divided at the points of intersection into eight equal arcs.