Mathematical Department.

BROWN UNIVERSITY, PROVIDENCE, RHODE ISLAND.

EXAMINATION FOR ADMISSION.

1. Reduce $\frac{1}{2}$ of 6% of $1{\cdot}05 \div \frac{3}{25}$ of $\frac{7}{10}$ to the simplest form.

Answer $= \dfrac{1 \times 6 \times 105 \times 25 \times 10}{2 \times 100 \times 100 \times 3 \times 7} = {\cdot}375.$

2. If 17 men can reap a field in 9 days, how long would it take to reap half of it, if 5 men refuse to work?

1 man could reap half the field in $\frac{1}{2}$ of 9×17 days.
12 men " " " " " " $\frac{1}{12}$ of $\frac{1}{2}$ of 9×17 dys. $= 6\frac{3}{8}$ dys.

3. A man bought 200 meters of cloth in France @ $16\frac{1}{4}$ francs per meter; he paid $12\frac{1}{2}$c. a yard for duty and freight, and sold it in Boston @ $\$4.62\frac{1}{2}$ a yard. What was his gain. (1 franc=19·3 cts.)

$\$4.62\frac{1}{2} - .12\frac{1}{2} = \$4.50$=actual selling price.
1 yard=36 in., ∴ 1 inch costs $\$4.50 \div 36 = \$\frac{1}{8}$
But 1 meter=39·37043 inches
∴ 1 meter sells for $\$\frac{1}{8} \times 39{\cdot}37043 = \$4{\cdot}921303$
Again 1 meter costs $16\frac{1}{4}$ f. $= 16{\cdot}25 \times 19{\cdot}3$ cents $= \$3{\cdot}13625$
gain on 1 meter $= \$4{\cdot}921303 - \$3{\cdot}13625 = \$1{\cdot}784053$
gain on 200 metres $= \$1{\cdot}784053 \times 200 = \$357{\cdot}0106.$

EDUCATION DEPARTMENT, ONTARIO, JULY EXAMINATIONS, 1884.

FIRST CLASS TEACHERS—GRADE C.

ALGEBRA.

Examiner—J. A. McLellan, LL.D.

Note—*Ten questions will constitute a full paper.*

1. Divide $x^5 - 5qx + 4r$ by $(x-m)^2$.
Find the relation between q and r, in order that the remainder may vanish.

2. When is any expression symmetrical with respect to two or more of the letters it involves?

(1). Find the square root of $3\{(a+b+c+d)^2 + (b+c+d+e)^2 + (c+d+e+a)^2 + (d+e+a+b)^2 + (e+a+b+c)^2 - (a^2+b^2+c^2+d^2+e^2)\}$.

(2). Simplify—

$$\frac{(a-b)^2-(b-c)^2}{a^2+ab-bc-c^2} + \frac{(b-c)^2-(c-a)^2}{b^2+bc-ca-a^2} + \frac{(c-a)^2-(a-b)^2}{c^2+ca-ab-b^2}.$$

3. Show that $(x-a)^{12} - x^6a^6 + (x^2-ax+a^2)^6$ is exactly divisible by $x^3 - 2ax^2 + 2a^2x - a^3$.

Find the factors of $(a^2-b^2)^5 + (b^2-c^2)^5 + (c^2-a^2)^5$.

4. Show how to extract the square root of a quantity of the form $a + b\sqrt{-1}$.

(1). Find the square root of $-3-\sqrt{-16}$.

(2). Show that one of the fourth roots of -64 is $2(1+\sqrt{-1})$.

5. Solve the equations $ax+by=c$, $a'x+b'y=c'$.

Interpret the result when $\dfrac{a}{a'} = \dfrac{b}{b'} = \dfrac{c}{c'}$

6. Solve the equations—

(1). $$\frac{ax+b}{a+bx} + \frac{cx+d}{c+dx} = \frac{ax-b}{a-bx} + \frac{cx-d}{c-dx}.$$

(2). $$\frac{x}{b+c} + \frac{y}{c-a} = a+b.$$
$$\frac{y}{c+a} + \frac{z}{a-b} = b+c.$$
$$\frac{z}{a+b} + \frac{x}{b-c} = c+a.$$

7. Find the relation between the roots and co-efficients of the equation $x^2+px+q=0$.

If the difference of the roots of the equation $x^2-(m-a)x+b^2=0$ is equal to the difference of the roots of the equation $x^2+(m-b)x+a^2=0$, show that $2m=5(a+b)$.

8. Prove that $\dfrac{a-b}{1+ab} + \dfrac{b-c}{1+bc} + \dfrac{c-a}{1+ca} = \dfrac{a-b}{1+ab} \cdot \dfrac{b-c}{1+bc} \cdot \dfrac{c-a}{1+ca}$

9. Solve the equations—

(1). $$x^3+y^3=a$$
$$xy(x+y)=b.$$

(2). $$\begin{cases}(x^3+x^2y+xy^2+y^3)(x+y)=a\\(x^3-x^2y+xy^2-y^3)(x-y)=b.\end{cases}$$

(3). $$\sqrt[3]{x+\sqrt{2}} + \sqrt[3]{x-\sqrt{2}} = \sqrt{2}.$$

10. Show that if the arithmetical and geometrical means of two quantities be given, the quantities themselves may be found, and give expressions for them.

(1). Sum the series $1 - \dfrac{2}{m} + \dfrac{1}{m^2} - \dfrac{2}{m^3} + \dfrac{1}{m^4}$—&c., *ad inf.*

(2). Show that the sum of n terms of the series $1+3+7+15+\ldots\ldots+(2^n-1)$ is $2^{n+1}-(n+2)$.

(3). Write down four terms of the series whose n^{th} term is $\dfrac{4n^2-1}{4n^2+1}$.

11. The number of combinations of $n+1$ things 4 together is 9 times the number of combinations of n things 2 together; find n.

12. Show that there are only $n+1$ terms in the expansion of $(1+x)^n$ when n is a positive integer.

(1.) Write down the 5^{th} term of $(1-x)^{-3}$

(2.) Write down the middle term of $(1+x)^{2n}$.

SOLUTIONS.

1.
$$\begin{array}{r|c|l} 1 & 1+0+0+0 & -5q+4r \\ +2m & 2m+4m^2+6m^3 & +8m^4 \\ -m^2 & -m^2-2m^3 & -3m^4-4m^5 \\ \hline & 1+2m+3m^2+4m^3 & \end{array}$$

∴ $5m^4=5q$, and $4m^5=4r$, *i. e.*, $q^5=r^4$.

2. See Teachers' Handbook of Algebra, p. 32.

(1). Following a we see that $3(3a^2+6ab)$ is part of the result ∴ by symmetry the whole result must be
$=3\{3(a^2+b^2+c^2+d^2+e^2)+6(ab+bc+cd+de)\}$
$=9(a+b+c+d+e)^2$ ∴ Sq. Rt. $=3(a+b+c+d+e)$.
—See Handbook, pp. 34, 35, 36.

(2). 1st fraction $=(a-2b+c)\div(a+b+c)$, hence by symmetry
2nd " $=(b-2c+a)\div(b+c+a)$
3rd " $=(c-2a+b)\div(c+a+b)$
∴ Sum $=$ 0 $\div(a+b+c)=0$.
—See Handbook, p. 119.

3. (*a*) $x^3-2ax^2+2a^2x-a^3=(x-a)(x^2-ax+a^2)$
Now if $x-a=0$, dividend=0, ∴ $x-a$ is a factor.
Again put $x^2-ax-a^2=0$, *i. e.*, $(x-a)^2=-ax$, or $(x-a)^{12}=a^6x^6$ and the dividend immediately vanishes, ∴ x^2-ax+a^2 is also a factor.—See Handbook, p. 48.

(*b*) For a^2, b^2, c^2, write x, y, z respectively and we get $(x-y)^5+(y-z)^5+(z-x)^5$, which $=5(x-y)(y-z)(z-x)$
$(x^2+y^2+z^2-xy-\&c.)$.
—See Handbook, p. 89, 12, and p. 64, 27–30.
∴ the factors are $5(a^2-b^2)(b^2-c^2)(c^2-a^2)$
$(a^4+b^4+c^4-a^2b^2-b^2c^2-c^2a^2)$
$=5(a+b)(b+c)(c+a)(a-b)(b-c)(c-a)$
$\{(a+b+c)(a-b-c)(a-b+c)(a+b-c)+(a^2b^2+b^2c^2+c^2a^2)\}$

4. Assume $x+y=\sqrt{(a+b\sqrt{-1})}$, ∴ $x-y=\sqrt{(a-b\sqrt{-1})}$,
∴ $x=$&c, $y=$&c, and $x+y=\frac{1}{2}\{2a+2\sqrt{(a^2+b^2)}\}^{\frac{1}{2}}$
$+\frac{1}{2}\{2a-2\sqrt{(a^2+b^2)}\}^{\frac{1}{2}}\sqrt{-1}$.

(1) $-1+2\sqrt{-1}$.

(2) $\sqrt{(-64)}=\pm 8\sqrt{-1}$. Take the upper sign and assume
$x+y=\sqrt{(0+8\sqrt{-1})}$, ∴ $x-y=\sqrt{(0-8\sqrt{-1})}$
∴ $x^2+y^2=0$, $x^2-y^2=8$, &c., $x+y=2+2\sqrt{-1}$.

5. Book-work. See Colenso, part II, §.48, 49. The equations are not independent of each other; the values of x and y are indeterminate.

6. (1) Transpose and add separately the two pairs of fractions and $ab(1-x^2)\div(a^2-b^2x^2)=cd(1-x^2)\div(c^2-d^2x^2)$
∴ $1-x^2=0$ is one solution, or $x=\pm 1$
and $ab\div(a^2-b^2x^2=cd)\div(c^2-d^2x^2)$ gives two more.

(2) Add the equations as they stand; strike out factor 2,