ANNEX A

THE NORDHAUS MODEL

A1. The pre-invention product market is competitive.
A2. Invention proceeds under conditions of certainty.
A3. Production in the industry is characterized by constant returns to scale.
A4. B(R) is the amount by which the unit cost of production is reduced by a process invention that is patented by a firm in the industry.
A5. B(R), called the "invention possibility function", is a concave function of R, where R is the number of units of inventive input employed. Also $B'(R) > 0$ and $B''(R) < 0$.

Let the industry face a linear demand function:

$$X(P) = \xi - \eta P \tag{1}$$

Then the pre-invention competitive price equals marginal and average cost of production. If C_0 is the cost per unit of output prior to invention and C_1 is the unit cost after the invention, then it follows that

$$B(R) = \frac{C_0 - C_1}{C_0}. \tag{2}$$

In this model, the maximum royalty which the inventor can charge for licensing all producers in the industry is equal to the total cost savings at the preinvention level of output X_0 given by the area of the rectangle C_0ABC_1, in Figure 1 above. So the inventor's royalty income is $(C_0 - C_1)X_0$ per period during the life of the patent. If C_0 and X_0 are normalized to 1, the royalty income of the inventor will be B(R) per period. After the expiry of the patent, price will decrease to P_1 and the rectangle C_0ABC_1 will be transferred from the inventor to consumers.

A lengthening of the patent term increases the present value of the royalties to the inventor, which in turn increases his incentive to invent. By investing additional resources in inventive activity, larger cost savings can be achieved. The area of rectangle C_0ABC_1 is increased as is the area of the triangle ABD, in Figure 1 above. However, the longer the patent term, the longer consumers have to wait to enjoy this triangle surplus. Thus, the optimal patent term requires a balancing of the loss of current consumers' surplus (which arises from extending the patent term) against the incentives for invention which will result in still larger consumer surplus in the future.