we get a linear equation for the determination of the water quantity, whereas the differential equation for the determination of the elevations also becomes linear.

The results thus obtained from these equations are naturally only approximate ones, as the overflowing quantities are introduced as too small. The computed values of z exceed the actual values. For practical purposes, this first approximation is generally sufficient, but we have no difficulties using the results of the first approximation for a second computation, drawing the tangent on that point of the overflow curve which corresponds to the maximum value of the elevation found in the first computation and repeating with these results the computation as before. We may use this second approximation in the first computation if instead of the value for the point in the curve, we take a somewhat smaller value, say, $u A s_e'$, when $u = 0.7$ to 0.8. We use the latter method in the following:

The overflow height, which gives an overflow of $u.A.s_e'$, is determined by

$$h_u = \left(\frac{3}{2}\,\frac{u}{\mu}\cdot\frac{s_e' A}{b'\sqrt{2g}}\right)^{2/3}$$

The proportional factor k for the linear variation of q is obtained by differentiation of q with respect to k. Therefore:

$$k = \frac{dq}{dh^1} = (\text{as } h' = h_u) = \mu b' \sqrt{2g\, h_u} = \sqrt[3]{3/2\, \mu^2 b'^2\, 2g\, u\, s_e' A}$$

k has the dimension $l^2.t^{-1}$, and the value of the abscissa e_1, which is the difference between the true elevation of the spillway crest and that obtained by approximation, is

$$e_1 = h_u - \frac{u.s_e'.A}{k} \qquad e_1 = \sqrt[3]{\frac{19}{12}}\left(\frac{u\, s_e' A}{\mu b' \sqrt{2g}}\right)^{2/3} \quad (84)$$

The values are easiest obtained graphically from the curve of the overflow quantities.

Therefore, with e_1 the height of the ideal spillway crest above the static level $n-n$, (that is $E = e' + e_1$) once determined, the computation of the first period of movement must be extended to the elevation E.

We obtain from the previously mentioned formulæ $z_e = E$ and s_e. These are initial values for the second phase, from which beginning we measure the time anew.

$$c = \frac{q}{A} = \frac{k}{A}(z - E) \quad \text{and therefore} \quad \frac{dc}{dt} = \frac{k}{A}\,\frac{dz}{dt}$$

and the equation 23 becomes

$$\frac{d^2z}{dt^2} + \left(\frac{1}{T_0} + \frac{k}{A}\right)\frac{dz}{dt} + \left(\frac{1}{T^2} + \frac{k}{A\,T_0}\right) z - E.\frac{k}{A.T_0} = 0$$

Introducing $y = z + m = z - \dfrac{E}{\dfrac{A\,T_0}{k\,T^2} + 1}$ and abbreviating

$$\frac{1}{T_0} + \frac{k}{A} = \frac{1}{T_0^1}; \quad \frac{1}{T^2} + \frac{k}{A\,T_0} = \frac{1}{(T^1)^2} \quad \text{we get}$$

$$\frac{d^2y}{dt^2} + \frac{1}{T_0^1}\,\frac{dy}{dt} + \frac{y}{(T^1)^2} = 0 \quad (85)$$

Corresponding to the investigations regarding the form of the general integral of this differential equation, we must investigate whether the difference

$$\frac{1}{(T_1^1)^2} = \frac{1}{(T^1)^2} - \frac{1}{(2\,T_0^1)^2}$$ is positive or zero, or negative,

which we obtain by substituting the values of $\frac{1}{T_0^1}$ and $\frac{1}{(T^1)^2}$

$$\frac{1}{(T_1^1)^2} = \frac{1}{T_1^2} + \frac{k}{2A}\left(\frac{1}{T_0} - \frac{k}{2A}\right)$$

by which formula the investigation mentioned may be carried out and the corresponding form of the general integral may be used.

The integration constants must be determined with the initial values

$$t = 0; \quad z_0 = E; \quad s_0 = s_e$$

The duration of the second period of movement is obtained from the equation for z, which is given by that value of t for which z becomes E once more. If that does not occur in a case of non-periodic movement, for instance, if the spillway crest lies below the level $n-n$, then the duration of the second period of movement is only limited by a new occurrence of any kind of outflow. Otherwise, the final values of the second period are the initial values of a following period, which must be handled the same as the first case. (Case A.)

The method of computation may be shown best by an example. Using the former example, we consider a spillway of 65.7 feet width, the crest of which is at the static level $n-n$. That is, for this assumption $e' =$ zero. The flow of 530 cubic feet per second is suddenly stopped.

From the results of case (A) we get

$$z_e' = 0; \quad t_e' = 106 \text{ sec.}; \quad s_e' = +\ .075 \text{ feet/sec.}$$

The velocity s_e' corresponds to the flow in the surge tank at the time t_e' of $q_e' = .075 \cdot 5380 = 404 \frac{\text{cu.ft.}}{\text{sec.}}$. For a spillway width of 65.7 feet and for $\mu = 0.6$, we get from the spillway formula $q \frac{\text{cu. ft.}}{\text{sec.}} = 208\, h' \sqrt{h'}$ and therefore for

$$u\,.\,q_e' = 282 \frac{\text{cu. ft.}}{\text{sec.}} \quad (u = .7)$$

an overfall height of $h' = 1.22$ feet and a proportional factor $k = 3/2\,.\,208\,.\,h_u^{1/2} = 345 \frac{\text{sq. ft.}}{\text{sec.}}$ and therefore as the distance of the ideal spillway crest from the static level $n-n$ because $e' =$ zero; $E = .410$ and with the results of case a for $z = E$

$$s_e = +\ .073 \text{ feet per second}$$

In order to determine which integral formula to use, we have

$$\frac{1}{(T_1^1)^2} = \frac{1}{.\,T_1^2} + \frac{k}{2A}\left(\frac{1}{T_0} - \frac{k}{2A}\right) = -\frac{1}{34.6^2}$$