ARTS DEPARTMENT.

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UNIVERSITY OF TORONTO,

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PROBLEMS (ALL THE YEARS).

Solutions by ANGUS MACMURCHY. (See C. E. MONTHLY for May-June, 1881.)

1. If a point O be taken in the interior of an equiangular triangle ABC, and if we drop perpendiculars OH, OI, OL on the three sides, the sum of these three perpendiculars is equal to the altitude of the triangle. (To be solved by geometry.)

Let OH, OI, etc., be perpendiculars on BC, CA, etc. Draw NON', MOM', POP' parallel to BC, AB and AC. Then by similar triangles, AD being the perpendicular on BC from A,

$$\frac{OM}{OH}=\frac{AB}{AD},\quad \frac{OM'}{OI}=\frac{AB}{AD},\quad \frac{OP}{OL}=\frac{AB}{AD}.$$

Now, $OM=BN$, $OM'=PA$, $OP=PN$,

$$\therefore \frac{OH+OI+OL}{BN+NP+PA}=\frac{AD}{AB},$$

$$\therefore OH+OI+OL=AD.$$

2. Find θ and ϕ from the equations,

$p\sin^4\theta-q\sin^4\phi=p$, $p\cos^4\theta-q\cos^4\phi=q$. Investigate whether θ, ϕ can both be real for any real values of p and q.

$$q-p=p(\cos^4\theta-\sin^4\theta)-q(\cos^4\phi-\sin^4\phi).$$
$$=p(\cos^2\theta-\sin^2\theta)-q(\cos^2\phi-\sin^2\phi).$$
$$\therefore p\cos^2\theta=q\cos^2\phi.$$
$$\therefore \cos\theta=\left\{\frac{q}{p}\left(\frac{q}{q-p}\right)\right\}^{\frac{1}{4}},\ \cos\phi=\left\{\frac{p}{q-p}\right\}^{\frac{1}{4}}.$$

If p and q be both positive or both negative, then θ and ϕ are real or imaginary according as $q><p$. If p be positive and q negative, or *vice versa*, then $p><q$, θ and ϕ are both imaginary.

3. If lines be drawn from the angles of a triangle ABC to the centre of the inscribed circle cutting the circumference in D, E, F, shew that the angles DEF of the triangle formed by joining these points are respectively equal to

$$\frac{\pi+A}{4},\quad \frac{\pi+B}{4}\text{ and }\frac{\pi+C}{4}.$$

Let O be the centre of the inscribed circle, then angle $EDF=\frac{1}{2}$ angle $BOC=$

$$\tfrac{1}{2}\left\{\pi-\frac{B}{2}-\frac{C}{2}\right\}=\tfrac{1}{2}\left\{\pi-\tfrac{1}{2}(\pi-A)\right\}=\tfrac{1}{4}(\pi+A).$$

4. Let $a_1\ a_2\ a_3\ldots$ be the lengths of the sides of a polygon $ABCD\ldots$ inscribed in a circle, $p_1\ p_2\ldots$ the lengths of the perpendiculars from any point P in the circle on the considered position. Then if the polygon be not reëntering, and if P be on the smaller arc cut off by a_1,

$$\frac{a_1}{p_1}=\frac{a_2}{p_2}+\frac{a_3}{p_3}+\ldots\ldots+\frac{a_n}{p_n}.$$

Take a triangle ABC inscribed in a circle, P any point on the arc AB, then letting fall perpendiculars $PP_1=p$ on $AB=a_1$, $PP_2=p_2$ on $BC=a_2$, $PP_3=p_3$ on $AC=a_3$, it is well known that the feet of these perpendiculars lie in a right line, and since the angle between any two lines equals angle between perpendiculars on those lines, we have since

$$\triangle PP_2P_1+\triangle PP_1P_3=\triangle PP_2P_3.$$
$$p_1p_2\sin B+p_1p_3\sin A=p_2p_3\sin C.$$