DF is equal to twice DH; therefore the rectangle DF, FH is double of DH, HF, *i.e.*, it is double of AH, HB contained by the segments of the base, made by DF which bisects the angle AFB. And AB was made equal to the given base.

Solutions by proposer, D. F. H. WILKINS, B.A., Math. Master, High School, Chatham.

194. If ABC be any plane triangle, and if the angle BAC be bisected by AD, meeting BC in D, then the rectangle contained by BC and CD is greater than, equal to, or less than the rectangle contained by BC and BD, according as the angle ABC is greater than, equal to, or less than the angle ACB.

Because angle B is greater than angle C,

$\therefore$ AC is greater than AB.

Cut off, from AC the greater, $AE=AB$, and join DE,

$\therefore$ the two triangles ADB, ADE are equal in all respects;

$\therefore$ $ED=BD$,

$\therefore$ angle ADE=angle ADB.

Now, angle CED is greater than angle ADE, and also greater than angle ADB; and angle ADB is greater than angle ACD;

$\therefore$ *à fortiori*, angle CED is greater than angle ACD,

$\therefore$ the side CD is greater than ED, *i.e.*, CD is greater than BD. Similarly, if angle C be greater than angle B, BD is greater than CD.

195. AB, CD are two chords in a circle, intersecting in any manner in a point P. Prove that the sum of the squares upon AP and PB is equal to the sum of the squares upon CP and PD, if the arc AC be equal to the arc BD.

Let $ACBD$ be a circle, AB and CD the chords as required. Then, since arc AC equals arc BD,

$\therefore$ arc AB equals arc CD,

$\therefore$ chord AB equals chord CD,

$\therefore$ square on AB equals square on CD,

$\therefore$ square on AP, PB and twice rectangle AP, PB equals square on CP, PD, and twice rectangle CP, PD.

Now, rectangle AP, PB equals rectangle CP, PD,

$\therefore$ square on AP, PB equals square on CP, PD.

196. Construct a triangle, given the three angles, any chord of the inscribed circle, and the ratio of this chord to the diameter of the same circle.

Let AB represent the given chord, and C and D the lines whose ratio equals that of the chord to the diameter of the inscribed circle. Find a fourth proportional to the three lines, and describe a circle with half of this as radius. Let EFG be the circle, O the centre. At O make angle EOF equal the supplement of one of the angles at the base, and angle EOG equal supplement of another of the given angles. Through E, F and G draw tangents, and these shall be the sides of the required triangle.

NOTE.—EO may be *any radius whatever.*

CAMBRIDGE UNIVERSITY EXAMINATION PAPERS, 1881.

MATHEMATICAL TRIPOS.

EUCLID AND CONICS.

1. The opposite sides and angles of parallelograms are equal to one another, and the diameter bisects them, that is, divides them into two equal parts.

In a convex polygon of an odd number of sides the middle points of all the sides are fixed, except one which describes a curve; prove that the angular points of the polygon describe equal curves.

2. If a straight line be bisected, and produced to any point, the rectangle contained by the whole line thus produced, and the part of it produced, together with the square on half the line bisected, is equal to the square on the straight line which is made up of the half and the part produced.

Shew how to draw, when possible, through two given points on the circumference of a circle, a pair of parallel chords, so that the rectangle under the chords shall be equal to a given square.