(c) The force of impact is reduced owing to the velocity lost by the water in passing over the surface of the vane. Some previous experiments on this subject afford data which will be used in approximating to the loss due to this cause.

(d) It is impossible, practically, to turn the water completely back on itself on account of the reaction which would take effect on the back of the succeeding vane.

Let u be the resolved velocity of the vane at P in the direction of motion of the jet.

v that of the jet.

The water strikes the vane with relative velocity $(v - u)$ and leaves it with relative velocity $c_w \ (v - u)$, where c_w is the ratio of final to initial relative velocities.

The force exerted on the vane in the direction of motion of the water is equivalent to the momentum of water destroyed per unit time, which is :—

$$\frac{m}{g} \left\{ (v - u) - c_w (v - u) \cos \delta \right\}$$

where δ is the angle of deflection of the water

$$\therefore F = \frac{m}{g} \ (v - u) \ (1 - c_w \, . \cos \delta)$$

Now $u = OP \, . \, \omega \, . \, \cos \theta$.

ω being the angular velocity of the wheel.

If the line of the jet cuts OL in N, and ON be called z,

$$\frac{ON}{QP} = \cos \theta$$

$$\therefore OP = \frac{z}{\cos \theta} .$$

$$\therefore u = \frac{z}{\cos \theta} \ \omega \, . \, \cos \theta$$

$$= z \, \omega$$

$$\therefore F = \frac{m}{g} \ (v - z\omega) \ (1 - c_w \ \cos \delta)$$

The moment of this force about the centre of the wheel is

$$Fz = \frac{mz}{g} \ (v - z\omega) \ (1 - c_w \ \cos \delta)$$

which is constant.

The work done per second is

$$F \, z \, w$$

$$= \frac{m \, z \, w}{g} \ (v - z \, w) \ (1 - c_w \cos \delta)$$

The energy available per second is mh,

h being the available head of water. The efficiency therefore is

$$\eta = \frac{z\omega}{gh} \ (v - z\omega) \ (1 - c_w \cos \delta).$$

If N be the number of revolutions per minute

$$\frac{N}{60} = \frac{\omega}{2\pi}$$

$$\therefore \eta = \frac{2 \pi N z}{60 \, gh} \left(v - \frac{2 \pi N z}{60} \right) (1 - c_w \ \cos \delta) \ \text{(I)}.$$

In calculating the value of v to be inserted in this expression it must be remembered that the velocity of the issuing jet is less than that theoretically due to the head.

In practice there will be a reduction of velocity due to two causes :

(1) Resistance of pipe line.

(2) Loss in discharge from the nozzle.

If l be the length of pipe,

d the diameter,