$$\frac{Fl}{4} = \frac{SI}{v} \quad \text{(Within the elastic limit)}$$

$$\text{or} \quad S = \frac{Flv}{4I} \quad (4)$$

Where S = fibre stress desired,

$\frac{I}{v}$ = section modulus of the beam,

l = length of the beam.

The problem has now resolved itself down to finding the value of F in equation (4), all the other terms of the equation being known. This in turn takes us back to equation (3), in which all the terms are known except the acceleration $\frac{d^2s}{dt^2}$.

A double differentiation of the deflection-time curve yields the acceleration.

The machine used for making the tests was a Hatt-Turner drop testing machine, having a maximum capacity of 72-inch drop, in use by the United States Forest Products Laboratory at Madison, Wis. It is provided with a cylinder which is rotated at a relatively high speed and has wound around its surface a metalized sheet for receiving the impression of the stylus fixed to the descending tup. A zero or base line is first drawn on the sheet, giving the position of the tup when it rests upon the beam as a static load. An electric contact releases the tup when it has been drawn up the desired height, and a tuning-fork record gives the scale of the abscissa in seconds per inch. The latter record was taken to serve as a check on the theoretical velocity attained by the tup at the instant of striking.

Deflections must be measured from a base line of no load, so that it was necessary to compute first the distance up from the line drawn with the tup resting on the beam to the position of no deflection, by the equation

$$y_1 = \frac{1}{48} \frac{W_t l^3}{EI}$$

A typical deflection-time curve as obtained in the tests is shown in Fig. 1 (*a*). The differential or velocity curve of (*a*) is shown as (*b*) immediately below, and the second differential or acceleration curve is shown at (*c*). The dashed horizontal line represents the base line drawn on the paper while mounted on the cylinder with the tup resting on the beam, and y_1 is the static-load deflection computed according to the method mentioned. Deflections are likewise distances of descent, s, of the tup, except at the start when the inertia of the beam has not been overcome. Up to the point where the curve crosses the time axis, the tup is falling freely under the influence of gravity, the velocity increasing in the downward direction proportional to the time, and the acceleration remaining constant, being equal to g, the acceleration of gravity. At m (neglecting inertia at the beam) the curvature changes, indicating a change in direction of the acceleration from negative to positive values. At r failure occurs and the acceleration curve shows a sudden drop. Rupture is not yet complete, but proceeds up to the point n. Here again there is a change in curvature of the space-time curve, signifying a change in the direction of the acceleration, which again becomes negative; and thereafter the body falls through the action of gravity alone.

Accelerations being the quantities desired, it is necessary to go through this process of differentiation for each curve, drawing first the velocity-time curve and then its differential by finding the slopes at a series of points on each curve, and plotting these slopes as ordinates to a new base line. This was done mechanically by means of the author's differentiating machine.

After computing the scales of each of the curves, we are in a position to draw the load or force-deflection curves for impact. By dividing the time axis into small units, finding the acceleration corresponding to each time interval, substituting in equation (3) to get the effective centre force F, and then scaling the deflections corresponding to each of these accelerations (hence forces), and plotting these values of force and deflection on co-ordinate axes, we have the desired force-deflection curve. A typical curve obtained in this manner is shown in Fig. 2. The beam was thoroughly air-dried long-leaf pine of a 2 x 2-inch section and span of 44 inches. The point of failure, as will be noted, is very pronounced, followed by continued deflection, in which the fibres that were not completely ruptured at the first failure are torn or crushed progressively.

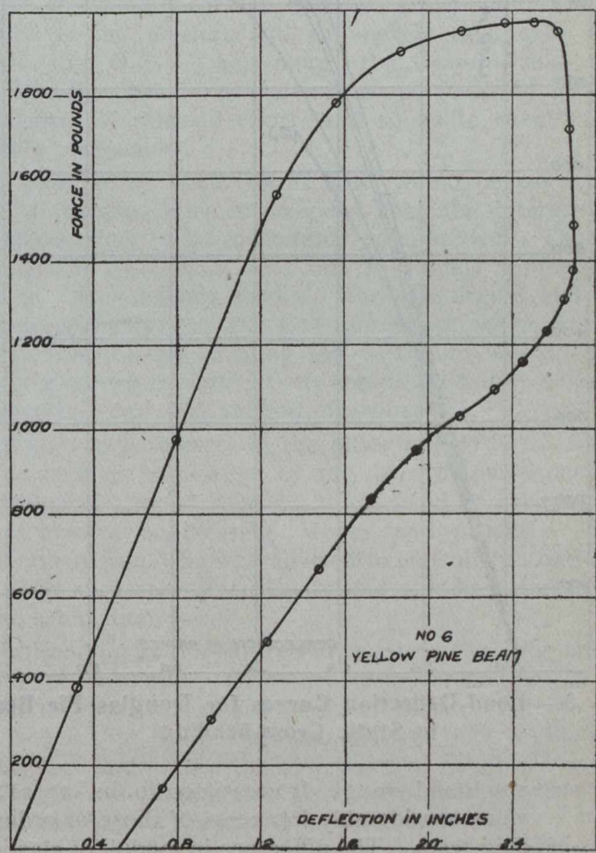

Fig. 4.—Impact Force-Deflection Curve for a Long-leaf Yellow Pine Beam Fractured but Retaining Considerable Resilience. The Circles Space Off Equal Time-intervals

Figs. 3 and 4 show the curves for two beams not completely ruptured, each retaining sufficient resiliency to send the tup back after the maximum deflection had been reached. The beam of Fig. 3 is peculiar in that initial failure was followed by an increase of deflection in which the load remained practically constant, indicating that the fibres failed at a rate just sufficient to balance the increase in force due to increase in deflection. At maximum deflection the total available energy had been consumed, and almost simultaneously there occurred a second failure. But the tup had already reversed its motion, as indicated by the returning curve. The latter phenomenon occurred more strikingly in the beam whose curve is shown in Fig. 4. The circles distributed along this curve are spaced at