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ANALYSIS OF STATICALLY INDETERMINATE FLAT ARCHES

STUDIES OF THE TWO-HINGED ARCH AND ARCH WITHOUT HINGES—A UNIFORMLY DISTRIBUTED LIVE LOAD ASSUMED OVER PARABOLIC STRUCTURE.

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IT is assumed in this paper that the arches possess a vertical axis of symmetry, have a rise less than 1/7 to ⅛ of the span, and that the supports are at the same height. Further, the computation is based on the assumption that the live load can be considered as uniformly distributed as for passenger bridges, floors, roofs and similar structures; that the arch takes the form of a parabola, and that the moments of inertia do not vary significantly.

The results arrived at are also applicable to circular arches with sufficient accuracy, while for arches with considerable differences in the cross-section at various points the general formulæ for statically indeterminate structures must be applied. The formulae given here will, however, for this case, as a rule, serve as a useful guide for the determination of trial dimensions.

The types which will be considered are: (1) the arch with two hinges at the springing lines, and (2) the arch

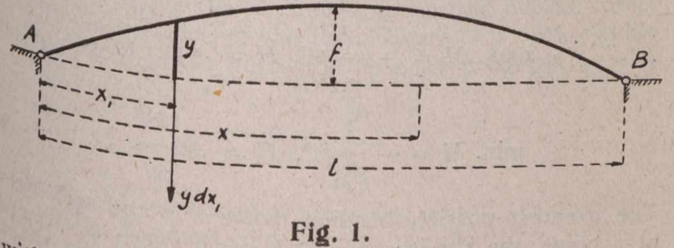

Fig. 1.

without hinges. The method followed in their analysis is the one developed by Professors Müller-Breslau and Ostenfeld.

The Two-Hinged Arch.—This arch has one statically indeterminate quantity and as such is chosen the horizontal pressure. The vertical components of the reactions are directly determinable by the statical equations and therefore not influenced by the dimensions of the cross-section of the arch nor by a slight yielding of the supports.

The equation of the line of influence of the horizontal pressure is $X = \frac{\delta_{ma}}{\delta_{aa}}$; δ_{ma} being the deflections of the various points, m, of the centre line of the arch, caused by a horizontal pressure of unity acting as sole load on the statically determinate auxiliary system. This system is, on account of the choice of the statically indeterminate quantity already referred to, a curved beam with one fixed and one roller end. δ_{aa} is the movement of the roller end produced by the same load.

From the general theory for statically indeterminate structures it is known that the deflections δ_{ma} can be determined as bending moments occasioned by the so-called "v" forces, acting on a simply supported beam with the same span as the arch; taking into account the assumptions mentioned in the introduction the "v" forces will be

$$v = ydx,$$

where y is the ordinate to the centre line of the arch, measured from the line between the hinges. The values of y corresponding to the various abscissæ x are given by the equation of the parabola which forms the centre line of the arch,

$$y = \frac{4f}{l^2} \times (l - x),$$

f being the rise of the arch, l the span, and x the abscissa measured from a hinge. When these "v" forces act on a simply supported beam, they will produce reactions which are equal to half the area between the arch and the line connecting the hinges, or ⅓fl; whence the bending moment (= δ_{ma}) at the point with the abscissa x

$$\delta_{ma} = \tfrac{1}{3}flx - \int_0^x ydx_1\,(x - x_1) \quad \text{(see Fig. 1.)};$$

and by application of the equation of the parabola

$$\delta_{ma} = \tfrac{1}{3}flx\left[1 - 2\left(\frac{x}{l}\right)^2 + \left(\frac{x}{l}\right)^3\right]$$

or

$$\delta_{ma} = \tfrac{1}{3}fx\,(l - x)\left[1 + \frac{x}{l} - \left(\frac{x}{l}\right)^2\right].$$

The denominator δ_{aa} of the expression for the statically indeterminate quantity is

$$\delta_{aa} = \int_0^l y^2 dx + li^2$$

where i is the constant radius of inertia.