Regarding the roots which do not apply see Colenso's Algebra, Part II., Section 43.

4. Put $S = 1^2 + 2^2 + 3^2 +$ etc. $+ n^2$. If $n = 0$ $S = 0$. $\therefore n$ is a factor of S.

Assume $1^2 + 2^2 + 3^2 +$ etc. $+ n^2 = An + Bn^2 + Cn^3 +$ etc., for all values of n.

$\therefore 1^2 + 2^2 + 3^2 +$ etc. $+ n^2 + (n+1)^2 = A(n+1) + B(n+1)^2 + C(n+1)^3 +$ etc.

Subtracting, $(n+1)^2 = A + B(2n+1) + C(3n^2+3n+1) +$ etc.

Equating coefficients $1 = 3C$, $2 = 2B + 3C$, $1 = A + B + C$

Whence $A = \frac{1}{6}$, $B = \frac{1}{2}$, $C = \frac{1}{3}$

And $S = \frac{1}{6}n + \frac{1}{2}n^2 + \frac{1}{3}n^3 = \frac{1}{6}n(n+1)(2n+1)$

5. (1) Book-work.

(2) Let $S = a + (a+b) + (a+2b) +$ etc. $+ \{a + (n-1)b\}$

Reversing, $S = \{a + (n-1)b\} + \{a + (n-2)b\} +$ etc. $+ (a+b) + a$

Adding, $2S = 2a + (2a + nb) + (2a + nb) +$ etc., n terms

$= n(2a) + (n-1)nb$

i.e., $S = \frac{n}{2}\{2a + (n-1)b\}$.

6. (1) Let $S = 1 + 3x + 5x^2 +$ etc. $+ (2n-1)x^{n-1}$

$\therefore Sx = x + 3x^2 +$ etc. $+ (2n-3)x^{n-1} + (2n-1)x^n$

i.e., $S(1-x) = 1 + 2x + 2x^2 +$ etc. $+ 2x^{n-1} - (2n-1)x^n$

adding and subtracting x^n

$= (1 - x^n) + 2x(1 + x^2 + x^2 +$ etc. $+ x^{n-2} + x^{n-1}) - 2nx^n$

$= (1 - x^n) + 2x\dfrac{1-x^n}{1-x} - 2nx^n$

$\therefore S = (1-x^n)\left\{\dfrac{1}{1-x} + \dfrac{2x}{(1-x)^2}\right\} - \dfrac{2nx^n}{1-x}$.

$= \dfrac{(1-x^n)(1+x)}{(1-x)^2} - \dfrac{2nx^n}{1-x}$

$= \dfrac{(1+x) - (2n+1)x^n + (2n-1)x^{n+1}}{(1-x)^2}$

If the series were infinite we see that $S = \dfrac{1+x}{(1-x)^2}$

(2)

1st	1
2nd	2+3
3rd	4+5+6
4th	7+8+9+10
etc.	etc.
nth	$\left(1 + \frac{n(n-1)}{2}\right) + \left\{2 + \frac{n(n-1)}{2}\right\} + \left\{3 + \frac{n(n-1)}{2}\right\}$ + etc. (n terms) $+ \left\{n + \frac{n(n-1)}{2}\right\}$.

Arranging as above, we see that the first term of each group = (sum of all numbers in left hand column except its own) + 1

$\therefore$ the nth group is as above, and

Sum of nth group $= \left\{n + 1 + n(n-1)\right\}\dfrac{n}{2} = \dfrac{n(n^2+1)}{2}$.

Sum of 1st n groups $= 1 + 2 + 3 +$ etc. $\left[\dfrac{n(n+1)}{2} \text{ terms}\right]$

$+ \cdots\cdots \dfrac{n(n+1)}{2}$

$= \left\{1 + \dfrac{n(n+1)}{2}\right\}\left\{\dfrac{n(n+1)}{2}\right\}\frac{1}{2}$

$= \left\{\dfrac{n(n+1)}{2} + \left(\dfrac{n(n+1)}{2}\right)^2\right\}\frac{1}{2}$

Again,

1^3	= 1	+ 0
2^3	= 2+3	+ 1+2
3^3	= 4+5+6	+ 3+4+5
4^3	= 7+8+9+10	+ 6+7+8+9
5^3	= 11+12+13+14+15	+ 10+11+12+13+14
etc.^{3}	= etc.	+ etc.
n^3	= etc.	+ etc.

Sum $=$ (sum of n groups) + (sum of n groups) − (last term)

$= \dfrac{n(n+1)}{2} + \left\{\dfrac{n(n+1)}{2}\right\}^2 - \dfrac{n(n+1)}{2}$

$= \left\{\dfrac{n(n+1)}{2}\right\}^2$

In a similar way we might infer the sum $1^2 + 2^2 + 3^2 + \ldots\ldots n^2$.

$1^2 = 1$, $2^2 = (2+3) - (1)$, $3^2 = (4+5+6) - (1+2+3)$, $4^2 = (7+8+9+10) - (3+4+5+6)$ etc.

7. (1) Book-work.

(2) 3 sets + 10 books = 13 things, which give $|\underline{13}$ variations

1st set gives $|\underline{5}$, 2nd gives $|\underline{3}$, and 3rd gives $|\underline{2}$ variations

$\therefore$ on the whole we have $|\underline{13}$, $|\underline{5}$, $|\underline{3}$, $|\underline{2}$ variations.

If each set may be arranged from either end, then each set will give two arrangements instead of one as above,

i.e., $2|\underline{5}$, $2|\underline{3}$ and $2|\underline{2}$, and the total number will be $|\underline{13}$, $|\underline{5}$, $|\underline{3}$ $\times 2^4$.

8. Altogether there are $4n+2$ points, or deducting A and B, $4n$ points. Each line parallel to AB will contain 4 points.

Take any point in the first circle and join it with two points in the second circle. This may be done in $\dfrac{2n(2n+1)}{|\underline{2}}$ ways.

Hence, for the whole $2(2n+1)$ points the number of possible triangles is $2(2n+1)\dfrac{2n(2n+1)}{|\underline{2}}$. But as there are n lines parallel, $4n$ triangles will vanish.

Hence, total number of triangles $= 2(2n+1)\dfrac{2n(2n+1)}{|\underline{2}} - 4n$

$= 2n(2n+1)^2 - 4n$.

9. Book-work.

10. (1) The $(r+1)$th term of $(1-x)^{-\frac{p}{q}}$

$(-1)^r \dfrac{p(p+q)\ldots\{p+(r-1)q\}}{|\underline{r} \cdot q^r}x^r$

Hence, for $(1-x)^{-\frac{3}{2}} = (-1)^r \dfrac{3 \cdot 5 \cdot 7 \cdots\cdots (2r+1)}{|\underline{r} \cdot 2^r}x^r$

Multiply numerator and denominator of coefficient by $|\underline{r}$ and 2^r.

it becomes $= \dfrac{|\underline{2r+1}}{(|\underline{r})^2} \cdot \dfrac{1}{2^{2r}}$.

(2) We have $a_0 = 1$ $a_1 = \dfrac{n}{1}$, $a_2 = \dfrac{n(n-1)}{|\underline{2}}$, $a_3 = \dfrac{n(n-1)(n-2)}{|\underline{3}}$

$\therefore \dfrac{a_1}{a_0} = n$, $\dfrac{2a_2}{a_1} = n - 1$, $\dfrac{3a_3}{a_2} = n - 2$ etc.

$\therefore S = n + (n-1) + (n-2) + \ldots n$ terms $+ 2 + 1 = \dfrac{n(n+1)}{2}$.

ANSWERS TO CORRESPONDENTS.

We have received from W. Braithwaite, Unionville, the following proposed solution of No. 4 in the First-Class Arithmetic, of which solutions were given in this department last month:

$90 due in 40 days.

$90 " 101 "

$\therefore$ $180 " $70\frac{1}{2}$ " (equated time).

Disct. on $180 for $70\frac{1}{2}$ " $= \frac{2}{100}$ of $180 = $$3\frac{3}{5}$.

" 180 " 365 " $= 3\frac{3}{5} \times \dfrac{365}{70\frac{1}{2}} = \dfrac{2628}{141}$

Or Int. on $176\frac{2}{5}$ " 1 year $= \dfrac{2628}{141}$

$\therefore$ " 100 or per cent = $10.56+

This is shorter than our solution but is not strictly accurate. Like several published solutions of this problem, it assumes interest equal to discount, viz., that the interest on $90 paid $30\frac{1}{2}$ days after it is due = the discount on $90 paid $30\frac{1}{2}$ days before it is due. It is doubtful whether a candidate for First-Class would receive full marks for an answer only approximately correct. In this particular question the sums are so small and the times so short that the difference between mathematical discount and bank discount is only very small, yet the principle of putting interest = discount is scarcely accurate, though it saves some labor in the calculation.

General Information.

An Electric Railway from Berlin to Lichterfeld has been successfully opened. The rails are insulated from the earth by wooden sleepers, and are in electrical connection with a dynamo-electric machine worked by steam power at a station. A magneto-electric machine on the driving carriage or locomotive is so fixed and connected with the axle of one pair of wheels as to impart motion to it, the driving axle being severed electrically by introducing an insulated washer, and a current of electricity, passed along one rail to work the magneto-electric machine on the locomotive, returns by the other rail to the stationary machine on the ground. The rate of speed attained was eighteen miles an hour.