

osophers of method maintain that  $\$12 \div \$4$  gives *not* the mental conception of *times (three)*, but a thing, *viz.*, "three, four dollars." Others tell us that  $\$12 \div 3$  demands an "impossible operation," because 3 cannot be taken from  $\$12$ , but that  $12 \div 3$  (*abstract numbers*) is a perfectly feasible and valid operation, "because we have  $12 - 3 = 9$ ,  $9 - 3 = 6$ , etc." But if, as Kant long ago pointed out, the mere thought of the union of the numbers 5 and 4 does *not* give the conception of their sum, we clearly may conclude that the mere thought of the division of 12 by 3 does not give the conception of the quotient four. The relations involved in this operation must be supplied first of all by *intentions—i.e.*, by acts with things. And so we have to ask what are the intentions, the preliminary acts with things, that lead to the *conception* of these relations? Of course in division, as well as in all other mathematical operations, we work with the pure number symbols, and at last make the necessary concrete applications, *i.e.*, *interpret* the results. But these symbols must have definite meanings to begin with. Not only so. These symbols and the operations in which they are involved in any problem, *must be capable of interpretation at any and every step*. When, therefore, I am told that  $\$12 \div 3$  is impossible; but  $12 \div 3$  is easy and valid, because  $12 - 3 = 9$ ,  $9 - 3 = 6$ , etc., I am not satisfied. I demand an interpretation of these steps. I must have a concrete illustration of this abstraction. What do these subtractions mean? And if no interpretation is forthcoming, I protest against the substitution of empty abstractions and sounding symbols, for clear and definite ideas. Some begin with things—the concrete—and stay there; others begin with the abstract—empty symbols—and stay there. The true way is from the

concrete (things) to the abstract, and from the abstract back again to the concrete. The individual, the concrete, without the general—the abstract—gives not true knowledge; the general without a rich filling of the concrete is but an empty name.

#### A DIVISION FALLACY.

This brings me to notice the assertion criticized in the November article, *viz.*: that the divisor can never be an abstract number. It was shown in that article, that the divisor may be an abstract number, and that every step of the operation in such division is capable of a common-sense explanation. One vice of primary number teaching is to exalt things at the expense of thought; another is that the "methods" have little or no *continuity* with the child's already acquired experiences; they do not bring into clear and definite consciousness what the child has long been unconsciously doing for himself. When a philosopher tells us, *e.g.*, that  $\$12 \div 3$  is impossible, and that the problem can be solved only by using *abstract numbers*, he violates this principle of continuity. He makes the abstract precede the concrete, he implicitly teaches that the child cannot distribute twelve things into three equal groups, till he has been taught the process of abstract division! But the child has actually done the thing again and again. He does as the savage does, and as the race did before number symbols were invented. His practical solution of the problem, implicitly involving certain thought relations, prepares him for the arithmetical solution in which the thought relations become explicit; *prepares him*—that is, if the things that that nature and natural education have joined together are not put asunder by the arbitrary decree of the empiricist. A child, *e.g.*, is required to distribute a *whole* of things into