DEFINITION.

A polyhedron is *regular* when its faces are similar and equal regular polygons.

THEOREM 6. *There cannot be more than five regular polyhedra.*

This is proved by examining the number of ways in which it is possible to form a solid angle out of the plane angles of various regular polygons; bearing in mind that *three* plane angles at least are required to form a solid angle, and the sum the plane angles forming a solid angle *is less than four right angles.* XI. 21.

Suppose the faces of the regular polyhedron to be *equilateral triangles.*

Then since each angle of an equilateral triangle is $\frac{2}{3}$ of a right angle, it follows that a solid angle may be formed (i) by *three*, (ii) by *four*, or (iii) by *five* such faces; for the sums of the plane angles would be respectively (i) two right angles, (ii) $\frac{8}{3}$ of a right angle, (iii) $1\frac{2}{3}$ of a right angle;

that is, in all three cases the sum of the plane angles would be less than four right angles.

But it is impossible to form a solid angle of *six* or more equilateral triangles, for then the sum of the plane angles would be equal to, or greater than four right angles.

Again, suppose that the faces of the polyhedron are *squares.*

(iv) Then it is clear that a solid angle could be formed of *three*, but not more than three, of such faces.

Lastly, suppose the faces are *regular pentagons.*

(v) Then, since each angle of a regular pentagon is $\frac{6}{5}$ of a right angle, it follows that a solid angle may be formed of *three* such faces; but the sum of more than three angles of a regular pentagon is greater than four right angles.

Further, since each angle of a *regular hexagon* is equal to $\frac{4}{3}$ of a right angle, it follows that no solid angle could be formed of such faces; for the sum of three angles of a hexagon is equal to four right angles.

Similarly, no solid angle can be formed of the angles of a polygon of more sides than six.

Thus there can be no more than *five* regular polyhedra.