PROPOSITION 2.

THEOREM.

Join the points A and n ; the distance An shall be equal to the perimeter of the polygon of an infinite number of sides, or equal to the circumference of the given circle ABD, (fig. 4.) FIG. 7.

For with the distance An, from A as a center describe through the point n, the arc *n*nN, meeting the arc BD in the point *n*'nN', and CD in the point N. Also from the point C, as a center, with the distance Cn, describe through n, the arc *n*'nN' meeting the arc BC in the point *n*', and CD in the point N' ; because the curve line Bdef, ... hikD' is through intersections of equal arcs on the arcs BD and BC (Lem. 1,) the arc B*n*', must be equal to the arc B*n* ; but the point n is the point of ultimate intersection by construction on the curve line Bdef ... hikD' which is through intersections of the infinite series of arcs through the points 1, 2, 3, ... n, and through the infinite series of points 2', 3', 4', ... n ; hence the point n, must be on the arc nN (fig. 5.) Also the point n, is the point of ultimate intersection of the infinite series of points 1'', 2'', 3'' ... n' and of the infinite series of points 2', 3', 4' ... n', each ntersecting the curve line Bdef ... hikD' ; hence the point n must be on the arc n'N', (fig. 5), and consequently the distance An, must be equal to A*n* (fig. 5), and equal to the circumference of the circle ABD, (fig. 4) ; but the part BG of the curve line Bdef ... hikD' is the arc of a circle (Lem. 2, fig. 3,) having its center on the right line ABG, in the point L, hence the point n is on the arc of a circle ; and the distance An is the determinate length of the circumference of the circle ABD (fig. 4).

SOLUTION SECOND.

PROPOSITION 3.

THEOREM.

From the point D' with the distances D'*e* and D'*f*, and from B, with the distances Bk'' and B*i*'', describe the intersections a' and b', and through the points a' and b', draw the straight line a'b'n, meeting the circular arc Bd*ef* ... h*i*G, (Lem. 2), in the point n. The point n shall be on the intersections of the arcs nN, n'N, BG and the right line abn,—and the distance An, shall be equal to the length of the circumference of the given circle ABD, (fig. 4.) FIG. 7.

For the point k'', is the variation of k towards *i*, and the point *i*'' the variation of *i* towards h, upon the common intercepted arc dk, (Lem. 10) ; and in the same manner as demonstrated, (prop. 1), that the curve of intersections, described through the points y'' and p'' and r'' and q'' may be in one straight line with their chords, the arc BK must move towards BD, and BK' must move towards BC, (Lem. 10) ; then it is evident that the distance B*x*'' must come to be equal to By'', and Bq'' must be equal to Bp--and K'u must be equal to K's, and K'v must be equal to Kr, for their ultimate ratios are equal ; but this can only be possible on the common arc dk---and therefore, (prop. 1), it is only through the points of variation k'' and i'', and the points *e* and *f*, that the curve line through the intersections n'b' ... n, in this case, will come to be on the same straight line with its chord a'n ; but the point n is by construction the ultimate intersection of the line n'b' ... n on the curve line Bd*ef* ... h*i*kD' of the intersections described from B and D', through the infinite series of points *e*, *f* ... n, and of k'', *i*'' ... *n* on the common arc through the intersections, of the series of arcs 1K, and 1''K ; hence the point n, must be on the arc *n*N, and also on the arc *n*'N', and consequently on the intersection of *n*N and *n*'N' ; but the part Bd*ef* ... h*i*G, of the curve line Bd*ef* ... h*i*kD' is a circular arc, (Lem- 2.) Therefore the distance An, must be the determinate length of the circumference of the circle ABD, (Fig. 4.)

SOLUTION THIRD.

PROPOSITION 4.

THEOREM.

Let the straight line abn, be drawn through the points of intersection a and b, and n'b'n' drawn through the points a' and b', (prop. 3), intersecting each other in the point n. The distance An, shall be the determinate length of the circumference of the circle ABD, (Fig. 4.) FIG. 7.