Let h be the height to which the water is raised, measured from the level of the water in the well to the centre of the orifice of discharge, v the velocity of discharge through the orifice, and V the velocity of the orifice in its circular path, as in Art. 158. Then the work due to the centrifugal force must equal the work of raising the water through the height h, increased by the work stored in the water at efflux; therefore

$$\frac{WV^2}{2g} = Wh + \frac{Wv^2}{2g};$$

$$\therefore \quad v = \sqrt{V^2 - 2gh}, \qquad (1)$$

and

$$v - V = \sqrt{V^2 - 2gh} - V$$

[as in (2) of Art. 158].

Now the work applied per second to raise each lb. of water must equal the work in raising the water through the height h, increased by the work remaining in the water after leaving the machine. Hence

$$\text{applied work} = h + \frac{(v - V)^2}{2g}$$

$$= \frac{\left(V - \sqrt{V^2 - 2gh}\right) V}{g}. \qquad (2)$$

The useful work is h foot-pounds per second; therefore

$$\text{efficiency} = \frac{gh}{\left(V - \sqrt{V^2 - 2gh}\right) V} \qquad (3)$$

$$= 1 - \frac{gh}{2V^2} - \text{etc.}, \qquad (4)$$

which increases towards the limit 1 as V increases towards infinity. Neglecting friction, therefore, the maximum efficiency is reached when the pump has an infinitely great velocity of rotation, as in the case of the reaction wheel.