MATHEMATICS.

Solutions to the Problems in the January Number.

1. Draw a line longer than the width of the ruler; place the ruler across this line, inclining the end above the line to the right until the edges of the ruler pass through the ends of the line, and rule a line along the left edge; similarly, with the top to the left, rule along the right edge.

2. Proceed as above, only rule along both edges of the ruler. An isosceles triangle will thus be described on each side of the line, and the line joining their verticles will bisect the given line.

3. Let BAC be the ang.; lay an edge of the ruler along AB so that the other edge may cut AC and draw CD along the edge cutting AC in C; CD will be parallel to AB; similarly, draw BD parallel to AC; AD shall bisect the ang. BAC.

4. Take any two chords in the cir. not parallel; bisect each as in (3); the pt. where these bisecting lines meet is the cen.

5. By giving n the values 1, 2, 3, &c., we find that the spaces described in the successive intervals are as the Nos. 1, $2\frac{1}{2}$, $3\frac{1}{4}$, &c; if, therefore, f be the accel. during the 1st interval the spaces described are $\frac{1}{2}f$, $\frac{5}{4}f$, $\frac{13}{8}f$, &c. Now, since the space in the 2nd interval is $\frac{5}{4}f$, and since f of this is due to the velocity acquired in the first interval, therefore the remaining $\frac{1}{4}f$ is due to the accel. in the 2nd interval; hence this accel. is $\frac{1}{2}f$. Similarly the accel. during the 3rd interval is $\frac{1}{4}f$, &c. If, then, the vel. acquired at the end of the 1st interval be v, there will be added to this in the second, third, &c., intervals, $\frac{1}{2}v$, $\frac{1}{4}v$, &c., respectively, so that the vel. finally becomes $(1+\frac{1}{2}+\frac{1}{4}+\text{\&c.})v$.

6. When the engine has gone 3 ft. its vel. is 30 ft. per sec.; but since the mass is now increased to $\frac{3}{2}$ M, the vel. is reduced to 20 ft. per sec., and the accel. to 100; therefore the vel., after passing over the next 3 ft. ($v^2 = u^2 + 2fs = 400 + 2 \times 100 \times 3 = 1000$), is $10\sqrt{10}$; the mass being now increased to $\frac{7}{4}$ M, the vel. reduces to $\frac{60}{7}\sqrt{10}$, and the accel. to $\frac{4}{7}$ of 150; hence the vel. after the next 3 ft. is $\frac{60}{7}\sqrt{17}$; and since the mass is now increased to $\frac{15}{8}$ M, the vel. becomes $8\sqrt{17} =$ 33 nearly.

7. After the lowest particle has fallen a ft., its vel. is $\sqrt{2ag}$; the mass is now increased to $2a$, and hence the vel. becomes $\frac{1}{2}\sqrt{2ag} = \sqrt{\left\{ag\frac{(2-1)(2\times2-1)}{3\times2}\right\}}$; after falling a ft. farther the vel. acquired is $\sqrt{\frac{5}{2}ag}$, so that the third particle starts with a vel. $\sqrt{\frac{10}{9}ag} = \sqrt{\left\{ag\frac{(3-1)(2.3-1)}{3.3}\right\}}$, etc; hence the vel. with which the n^{th} begins to move is $\sqrt{\left\{ag\frac{(n-1)(2n-1)}{3n}\right\}}$.

8. $\dfrac{a+b}{1-ab} = \dfrac{c+d}{cd-1}$

$\therefore acd + bcd - a - b = c + d - abc - abd$

$\therefore acd + bcd + abc + abd = a + b + c + d$

$$\therefore \frac{a+b+c+d}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}} = \frac{(a+b+c+d)abcd}{bcd+acd+abd+abc}$$

$$= \frac{a+b+c+d}{a+b+c+d}abcd = abcd.$$

PROBLEMS.

9. If a slip of paper with parallel edges be tied in a common knot, and the edges drawn tight without crumpling, a regular pentagon will be formed.

10. A cistern is kept constantly supplied with water. Supposing that it is full at first,