SCHOOL WORK.

MATHEMATICS.

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PROBLEMS FOR JUNIOR MATRICULATION, 1887.

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(*Continued from October No.*)

9. Eliminate l, m, n, from the equations $(A) a^2 l^2 + b^2 m^2 + c^2 n^2 = a_1{}^2 l + b_1{}^2 m + c_1{}^2 n$ $al = bm = cn$ and $(B)\ l^2 + m^2 + n^2 = 1$.

9. Let $al = bm = cn = k$; $\therefore\ l = \frac{k}{a}$, etc.

Substitute in (A)

$$\therefore k^2\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)=\frac{a_1{}^2}{a}+\frac{b_1{}^2}{b}+\frac{c_1{}^2}{c}$$

again from (B) $k^2\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right)=1.$

$$\therefore \frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\left(\frac{a_1{}^2}{a}+\frac{b_1{}^2}{b}+\frac{c_1{}^2}{c}\right)\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right).$$

10. The number of ways in which r things may be distributed among $n+p$ persons, so that certain n of the persons may have one at least, is $(n+p)^r-(n+p-1)^r-n(n+p+1)^r$ $+\frac{n(n-1)}{\underline{|2}}(n+p-2)^r$ + etc.

10. The number of ways in which $n+p$ people $(A, B, C, \ldots)$ may receive r things is $(n+p)^r$. Next suppose A always receives one, then all ways in which r things could be given to the remaining $n+p-1$ would be excluded, *i.e.*, $(n+p-1)^r$ ways. Similarly with $B, C, \ldots$ so that if some one of n persons must receive one, $n(n+p-1)^r$ would be excluded from the $(n+p)^r$ ways, *i.e.*, $(n+p)^r - n(n+p-1)^r$. Next, when some two of the n get one each at least, we must exclude from the $n(n+p-1)^r$, unaccountable cases those in which one person (B, for example) always receives one, *i.e.*, $(n+p-2)^r$ Now since we have $\frac{n(n-1)}{\underline{|2}}$ combinations of n persons two at a time, we must exclude $\frac{n(n-1)}{\underline{|2}}(n+p-2)^r$ ways from the $n(n+p-1)^r$ exclusion when one person gets one, *i.e.*, $(n+p)^r - n(n+p-1)^r$ $+\frac{n(n-1)}{\underline{|2}}(n+p-2)$, and so on until every one of the n persons receives one.

11. If $l\cos(\theta-\beta) - m\cos(\theta-a) = n$, show that $l\sin(\theta-\beta) - m\sin(\theta-a)$ $=\sqrt{l^2+m^2-n^2-2lm\cos(a-\beta)}$.

11. Let $\theta-\beta=\phi$, and $\theta-a=\psi$. By squaring we get $l^2\cos^2\phi + m^2\cos^2\psi - 2lm\cos\phi\cos\psi = n^2$; $\therefore l^2+m^2-n^2-(l^2\sin^2\phi + m^2\sin^2\psi - 2lm\sin\phi\sin\psi) - 2lm(\cos\phi\cos\psi + \sin\phi\sin\psi) = 0$; $\therefore l^2+m^2-n^2-2lm\cos(a-\beta) = l^2\sin^2\phi + \ldots$

$$=(l\sin\phi - m\sin\psi)^2. \quad \text{Q.E.D.}$$

12. If $\sin^{-1}\frac{x}{a} + \sin^{-1}\frac{y}{b} = \sin^{-1}\frac{c^2}{ab}$ then

$b^2x^2 + 2xy\sqrt{(a^2b^2-c^4)} + a^2y^2 = c^4$.

12. $\text{Sin}\left(\sin^{-1}\frac{x}{a} + \sin^{-1}\frac{y}{b}\right)$

$$=\frac{x\sqrt{b^2-y^2}}{ab}+\frac{y\sqrt{a^2-x^2}}{ab}=\frac{c^2}{ab},$$

square $\therefore x^2b^2 - 2x^2y^2 + a^2y^2$

$$+2xy\sqrt{(b^2-y^2)(a^2-x^2)} = c^4\ (A);$$

again $\cos\left(\sin^{-1}\frac{x}{a} + \sin^{-1}\frac{y}{b}\right)$

$$=\frac{\sqrt{(b^2-y^2)(a^2-x^2)}}{ab}-\frac{xy}{ab}=\frac{\sqrt{a^2b^2-c^4}}{ab},$$

substitute this in (A), we get

$$b^2x^2 + a^2y^2 + 2xy\sqrt{a^2b^2-c^4} = c^4.$$

13. The area of any triangle is to the area of the triangle formed by joining the points