The matter of industrial education is being taken up everywhere by the manufacturers. In St. Louis a number of the large manufacturers have arranged to give the boys in their factories the liberty of attending seven hours of instruction per week. The course will cover mathematics, mechanical machine design and draughting. The boys will receive pay for the time they are attending these classes. These men recognize that, although this training will make the boys more valuable to themselves, it will also increase their value to their employer.

Cassier's Magazine for March will be of great interest to the engineering profession, especially those interested in railway building and railway operation. They call their March number a Railway Number, and among the articles are contributions from the most authoritative writers on the continent. The track, the bridge, the steam locomotive, the running gear, the signalling system, the tunnel, the shops and special equipments are all covered by a series of very special articles. This railway number will be preserved as a reference number by engineers.

THE HYDROSTATIC CHORD*

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The "Hydrostatic Chord" is allied to the catenary, the parabola and the circle because all of these curves may be formed by a flexible, inextensible substance, supported at its two extremities and properly loaded. If the load is uniformly distributed with respect to a horizontal line joining the supports, the action of gravity will shape the supporting substance to form a parabola. If the load is uniformly distributed with respect to the curve itself a catenary is the result. If the load is applied by fluid pressure, irrespective of the direction of gravity, so that the pressure is of uniform intensity normal to the curve a circle is formed. If the load is applied by fluid pressure which varies according to the head or depth of water at any point, the curve resulting from this system of forces normal to the curve is a hydrostatic chord, which can easily be imagined as the curve which a flat canvas hammock would take if filled with water.

If the canvas were sewed together to form a closed curve and supported on end as a vertical cylinder the cross section would become circular under fluid pressure. If now the open ends of the cylinder were sealed with flexible bulkheads and it was tipped over on its side when completely filled with water the cross section would become a hydrostatic chord although it would still, theoretically, be a circle also until a drop of water were allowed to escape—that is, if the water be regarded as incompressible. Since the shell of the cylinder is assumed inextensible and a circle encloses a maximum area for a given perimeter, it follows that the mere act of tipping such a cylinder towards the horizontal position would immediately develop infinite stress in the enclosing membrane if the water could not partially escape. If one now imagines a hole pricked in the side of the cylinder on top, and connected with a vertical pipe or piezometer, some of the water would escape up into the pipe and at the same time the shape of the cylinder would take on the characteristics of a true hydrostatic chord. The water would continue to rise until the head above the top was just sufficient to hold the remaining water in the equilibrium shape. At this time the surrounding membrane would be in pure tension, equal throughout, and of finite value. The very fact that the tension must be constant in all portions of the shell furnishes an easy means of constructing its shape because the tension at any point is obviously measured by the product of the radius of curvature and the head at that point.

If more water now be poured into the piezometer pipe the shape of the cylinder will approach a circle; its vertical diameter will lengthen and its horizontal diameter will shorten. It would, however, never become a circle for a finite head of water. Conversely if water be drawn off, the shape would become more and more oblate until finally the membrane would collapse into a plane surface at zero head.

It seems apparent from the above discussion that a circle is not a natural shape for a pressure pipe lying on its side especially if its diameter is large as compared with the water pressure. No one would think of designing a vertical water tank with an elliptical cross section, and thus subject the shell to enormous unnecessary deforming stresses in its effort to become circular. And yet it has not been unusual to design concrete pressure pipes not only circular, but even to go to the other extreme and shape the section with its least radius of curvature at the top instead of at the haunches.

Thi: latter procedure might be compared to designing a stiff suspension bridge cable, say for the sake of illustration, of reinforced concrete, and shaping it like an ellipse with its long axis horizontal instead of the more natural shape in which the radius of curvature would decrease toward the centre instead of increasing as in the former case. Such a chord would, obviously, be so ridiculous that an example of it could not be found in practice. Instead, it would probably be designed of parabolic shape which would be perhaps as good a compromise as one could arrive at. If it were really stiff and unable to adjust its shape to changes of load, as, for example, when a moving load passed over the bridge, then more or less severe deforming stresses would be the result, but how much less these would be in the latter case than in the former!

Similarly, although it is impossible to design a stiff pipe of such shape that there would be no deforming stresses under the varying conditions of water pressure and back fill, and the constant weight of the shell itself, yet these stresses can be reduced to a minimum by adopting a form which lies midway between the ideal equilibrium shapes which the pipe tries to assume under the various water pressures to which it may be subjected. This matter is of so much practical importance in large conduits that it means the saving of perhaps one-half the material, concrete or steel, by properly designing the shape of the cross section.

It is not the purpose of this paper to take up the mathematics of the hydrostatic chord, nor to follow through the complications of a typical design. The element of judgment enters so largely into such a study that it is impossible to do it justice in a restricted space and time. For the benefit of those who may be sufficiently interested to follow up the subject a few general hints may be of service. It is well known that a circular cylindrical shell lying on its side has four nodes or points of contra-flexure, due to its own weight. These lie at points 50° 36′ 45″ and 146° 19′ 25″, respectively, from the vertical.

It can be demonstrated that the location of these nodes due to the weight of water within such a cylinder is the same. It can also be shown that the bending moments due to both causes are exactly proportional at all points of the arc, and may, therefore, easily be combined. The equilibrium shape which would sustain the existing water pressure without any tendency to deform can easily be plotted from the polar equation of the bending moments in a circle in terms of the angle of departure from the vertical, remembering that the radial intercept between the circle and the new curve at any point is a measure of the bending moment at that point, and

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*Read before the Toronto Branch of the Canadian Society of Civil Engineers.