Now $\frac{d\theta}{dt}=\Omega$, when $\theta=o$, so that

$$\frac{mk^2\Omega^2}{2}=HMl(1-\cos\delta)$$

$$=2HMl\sin^2\frac{\delta}{2}$$

But HMl is a constant so long as the same galvanometer is used, and no alteration is made in its position, or otherwise. Therefore, $\sin^2\frac{\delta}{2}$ is a measure of the work done on the needle by the instantaneous current.

To find the time of oscillation of a magnetic needle placed under the influence of a uniform magnetic field.

The effect of the magnetic field on the needle, is to apply a force to each pole, parallel to the position of equilibrium. These two forces form a couple, and if θ be the angle of deflection of the needle at any time t, measured from the position of equilibrium, and l the distance between the poles, $Fl\sin\theta$, or $Fl\,\theta$, since θ is a small angle, is the moment of this couple. Hence the equation of motion is

$$mk^2\frac{d^2\theta}{dt^2}=-Fl\,\theta$$

Integrating and remembering that $\frac{d\theta}{dt}=o$, when $\theta=\delta$.

$$mk^2\left\{\frac{d\theta}{dt}\right\}^2=Fl(\delta^2-\theta^2)$$

Whence it will be found that the time of an oscillation is

$$2t=\pi\left\{\frac{mk^2}{Fl}\right\}^{\frac{1}{2}}$$

or, writing t for the time of an oscillation,

$$t^2=\frac{\chi}{F}$$

where χ is a constant.