(4) Prove the formulæ

(*a*) $\text{Sin. } 2\text{ A} = \dfrac{2 \text{ tan. A}}{1 + \text{tan.}^2\text{A}}$

(*b*) $\text{Sin.}^2\text{ A} - \text{sin.}^2\text{ B} = \text{sin. (A + B) sin. (A} - \text{B)}$

(*c*) $\text{Tan}^2\text{ A} = \dfrac{1 - \text{cos. } 2\text{ A}}{1 + \text{cos. } 2\text{ A}}$

(*d*) $\text{Tan } \frac{1}{2}\text{ (A + B)} = \dfrac{\text{sin. A} + \text{sin. B}}{\text{cos. A} + \text{cos. B}}$

(5) In a triangle, A B C, whose sides are a. b. c., and perimeter 2 s, prove that

$$\text{sin. } \tfrac{1}{2}\text{ A} = \sqrt{\frac{(s-b)(s-c)}{bc}}$$

(6) From a boat I observe that the elevation of the top of a tower on a cliff is 21° 20′, and rowing directly towards the tower for 500 yards, I now observe the elevation of the top and bottom of the tower to be 49° 3′, and 41° 20′, respectively. What is the height of the cliff and of the tower?

(7) In a triangle A B C, given that A B = 50 ft. B C = 20 ft. and angle A B C = 20° 15′, find the remaining side and angles.

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