

head or the two ears on his head is the number one; but ordinary infants do not have the shadow of such an idea; and yet the four-year old infant knows he has two eyes and two ears as certainly as the philosopher.

4. The theory that every number is a ratio excludes all *concrete numbers*. Every ratio is a quotient and every quotient is abstract, and hence every ratio is necessarily an *abstract number*. Take, for example, the concrete number 5 inches. It is clear that 5 inches is not a ratio, nor is the "5" in the expression "5 inches" a ratio. It is true that 5 times 1 inch = 5 inches, and that the "5" in the *first* member of the equation (5 times 1 inch) may be considered a ratio; but the first member of the equation *expresses a process*, and the second member (5 inches) is the *resulting number*, and this is not a ratio, but a collection of *concrete units*. No theory of number that excludes concrete numbers can be a true working theory for primary instruction in arithmetic. Concrete numbers have a large place in the child's experience, and they should have the first and the chief place in number instruction. It may be added that a concrete unit is not necessarily an object that can be seen or touched, or even imaged. It may be a period of time, the duration of silence, a power of the mind, an idea or thought, a feeling or a wish. Nor do all concrete numbers have a unit that is definite in consciousness. The unit is often as vague and indefinite as the number which it measures.

The above facts clearly show, as it seems to me, that the theory that all numbers are necessarily ratios is philosophically erroneous. If this conclusion be true, it follows that the basing of primary instruction in numbers on this theory is *an error in pedagogy*. The child's ideas of number do not involve the idea of ratio, and nothing can be gained by forcing the idea of ratio into early number processes. Further, since the ideas of numbers as collections of ones are acquired before the idea of product, and the idea of product before the idea of quotient; it seems to follow as a sound pedagogical principle that factor and ratio ideas and processes should be taught *after* the child has clear ideas of primary numbers and some skill in numbering objects, if not in combining and separating numbers. This early instruction in number should not deal too exclusively