We obtain the angle ψ from $tg\,\psi = -\dfrac{T}{T_0} = -\dfrac{1}{tg\,\phi}$ (79)

because $\dfrac{T_0}{\tau} = tg\,\phi$ and $\tau = T$.

According to the theory of the phenomena of resonance, the amplitude of the forced oscillation becomes of infinite value when the damping force is infinitely small. The latter would be the case, using our terms, when $T_0 =$ infinite with which value the amplitude of the above-mentioned equation also becomes infinite. The phase difference between force and movement has thereby to be equal to $\dfrac{\pi}{2}$, which also finds its expression in this equation (79).

2. Graphical Demonstration.—The graphical demonstration of the movement of the first phase is not difficult. The demonstration of the values

$$z_1 = R\,e^{-\frac{t}{2T_0}} \sin\left(\beta + \frac{t}{T_1}\right)$$

may be carried out in the same form as before by means of the projection of the logarithmic spiral. The demonstration of

$$z_2 = b\,.\sin\left(\psi + \frac{t}{\tau}\right) = b\sin\left(\psi + \frac{T_1}{\tau}\,.\,\frac{t}{T_1}\right)$$

is obtained by projection of a circle, constructed with the radius b in the polar system, where the initial vector is inclined against the axis of abscissæ with an angle ψ.

In this case a rotation of the initial vector of z_1 by an amount of 2π corresponds to a rotation of the initial vector of z_2 by an amount equal to $\dfrac{T_1}{\tau}\,2\pi$. (See Fig. 6.)

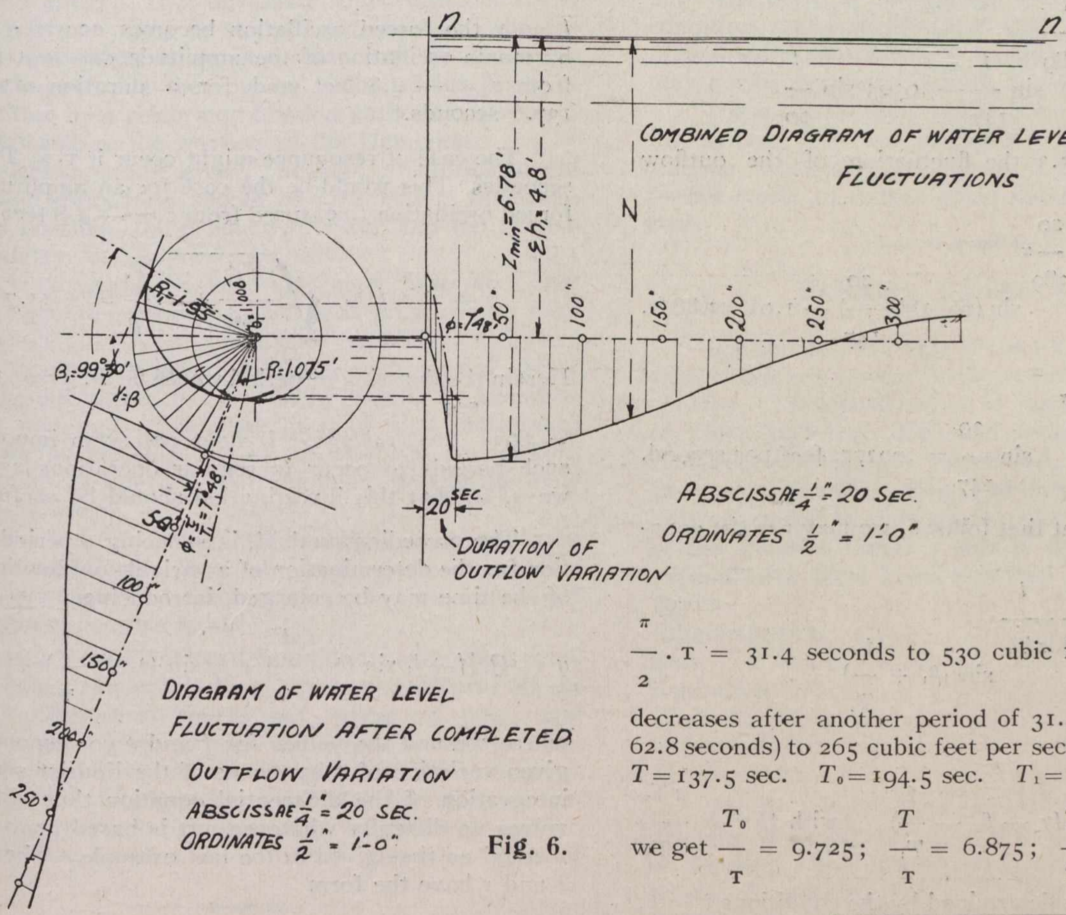

Fig. 6.

3. Analytical Example.—We use the same conduit and surge tank dimensions and assume the same friction conditions. As before, we take $\epsilon = 0.5$; with $f = 1$ and for $\tau = 20$ seconds we have an initial flow of 265 cubic feet per second which increases during the time $\dfrac{\pi}{2}\,.\,\tau = 31.4$ seconds to 530 cubic feet per second, and decreases after another period of 31.4 seconds (a total of 62.8 seconds) to 265 cubic feet per second. Therefore, with $T = 137.5$ sec. $T_0 = 194.5$ sec. $T_1 = 147$ sec. $h_1 = 9.6$ ft.

we get $\dfrac{T_0}{\tau} = 9.725$; $\dfrac{T}{\tau} = 6.875$; $\dfrac{T}{T_0} = .707$

$$b = -\epsilon f h_1 \frac{\sqrt{1 + \frac{T_0^2}{\tau^2}\left[\frac{T^2}{\tau^2} + \frac{T^2}{T_0^2} - 1\right]^2}}{1 + \frac{T^2}{\tau^2}\left[\frac{T^2}{\tau^2} + \frac{T^2}{T_0^2} - 2\right]} = -1.008 \text{ feet}$$

$$tg\,\psi = -\frac{T_0}{\tau}\,\frac{T^2}{\tau^2}\left[\frac{T^2}{\tau^2} + \frac{T^2}{T_0^2} - 1\right] = -457.1$$

ψ lies in the fourth quadrant and is equal to 270° 7′ 50″, which is practically 270°. With the latter value $\sin\psi$ becomes -1 and $\cos\psi = o$. We get

$$R\sin\beta = +b \qquad R\cos\beta = +\frac{T_1}{2T_0}$$

and therefore

$$tg\,\beta = \frac{2T_0}{T_1} = tg\,\gamma \qquad \beta = \gamma = 69^\circ\ 18'$$