philosophic genius, they may, like a fact in chemistry, when once discovered, be reproduced and applied by the dullest intellect. They could be thus easily attained if mathematical reasoning were a series of mere mechanical steps "passively" taken by the mind, But from what has been already advanced, we are justified in declaring this to be a groundless assumption—the utterance of an uncandid critic, or of a novice in the science. The discovery of new truths, or an original application of the old may have been a work of comparative ease to the man of superior genius. But the clear comprehension of the modes of investigation, and the complete appropriation of the discoveries, compel from the ordinary intellect the highest exertion of its powers. This will be corroborated by every student who has made himself master of any important branch of the science. The assertion that the works of the immortal Newton can be mastered by the exertion of a minimum of mental power, is too astounding a paradox to merit serious consideration. In the unquestioned judgment of mankind they stamp him as the philosopher *qui genus humanum ingenio superavit.* They stand conspicuous as the grandest monument of intellectual power that the world has ever seen, and shed a lustre on his age, before which the glories of all preceding times grow dim. They have established a greatness that does not vanish in the mists of years, but is carried onward down the stream of time with a splendor ever gathering from the triumphs of a distinction that can never die. They constitute not the transient and visionary philosophy of an epoch, but the creed of all time, and their author has become not the forgotten representative of a metaphysical sect, but the educator of the human race.

I may add further that in every mathematical training worthy of the name, the inventive powers which in their highest degree constitute genius, are called into exercise and fostered in teaching mathematics to the merest tyro. For in every rational training the solution of problems forms an important part—problems which are not mere repetitions of the type—questions given in illustration of principles--but so constructed as to test both the knowledge and the inventive powers of the pupil. This is true even in the simpler branches of the science. In elementary algebra, for example, a great variety of problems can be constructed to illustrate even the simple formulas in multiplication, which require for their solution no small degree of ingenuity. Of course, if problems are merely ceaseless repetitions of a certain type, their solution soon becomes as mechanical a process as repeating the multiplication table. But no mathematical teacher worthy of the name is ever guilty of such palpable cramming; and no mathematical examiner worthy of his trust will, by setting questions of this purely mechanical type, commit the serious error of encouraging a system of mathematical teaching which condemns the pupil to a minimum of thought. The thorough teacher and the competent examiner will so direct and control mathematical training as to expand and invigorate the same faculties of the mind, which are of closest kin to those of the greatest philosopher, and which in their highest degrees have produced the greatest discoveries in mathematical science.

4. Mathematics are of scarcely less importance in educating to an *accurate use of language and consequent skill in detecting the fallacies arising from its ambiguous use.*

Though words should be the passive subjects of the understanding, they sometimes, as it were, revolt from its authority, and create universal anarchy in the empire of thought. It is generally admitted that to inadequacy and ambiguity of words may be attributed a large portion of the errors which ensnare the understanding and impede its progress in the discovery of truth. Among "the four species of idols" which Lord Bacon has distinguished as "besetting the human mind," he ranks the *idola fori*- those which arise from the imperfection of words,—as the "most troublesome of all." He observes, "words still manifestly force the understanding, throw everything into confusion and lead mankind into vain and innumerable controversies and fallacies, hence the great and solemn disputes of learned men often terminate in controversies about words and names, in regard to which it would be better, imitating the caution of mathematicians, to proceed more advisedly in the first instance, and to bring such disputes to a regular issue by definitions." And Locke uses still stronger language in reference to the same subject, attributing to the incompleteness of words, almost all the errors that have obscured genuine knowledge and characterized the disputes of mankind. Though the latter may have stated the case somewhat too strongly, since it seems hardly possible that the solemn responsibilities of life should have been so generally sacrificed in a mere contest about words, it is nevertheless true as stated above that the incompleteness and ambiquity of words have proved a fruitful source of error, and a serious hindrance to the progress of knowledge. Hence the importance of being *accustomed* to the accurate use of words, and skilled in detecting the illusions lurking in their ambiguity. To this, we think, the study of mathematics eminently conduces.—Their language is precise and adequate, in consequence of the clear and distinct conceptions which they involve. No word is defective from inadequately representing the conception for which it stands: nor ambiguous from admitting anything extraneous; all are complete representatives of the things signified, and preclude the possibility of vitiating demonstration either in the admission of any foreign element, or the exclusion of any part of the case under consideration. Does not this constant and necessary accuracy in the use of words, habituate the mind to a corresponding accuracy of language in other departments of knowledge, and educate to skill in the detection of its fallacies? Or, as has been asserted, is mathematical science in consequence of this unerring exactness in its terms, utterly incapable of fortifying the mind against illusions from which it is itself exempt? Must we plunge at once into the reeling tempest of conflicting meanings to become accustomed to accuracy in the use of words? It seems evident that the necessary use of accurate forms of expression must tend to the formation of a *habit* of accuracy. Are we familiarized with the characteristics of the perfect by being first accustomed to imperfection? Are habits of certainty and precision in the use of language, best formed by our being first familiarized with its variable and ambiguous meanings? On the contrary, admitting the possibility of the formation of such habits by this mode of procedure, they could only be acquired from repeated experience in the illusions of language, and would consequently require the unnecessary expenditure of mental energy.

It is surely far better to enter upon any subject of investigation in which errors are likely to arise through the imperfection of language—not depending upon the successive corrections of erroneous results for ultimately creating habits of precision in expression—but already possessing such habits from the constant use of words characterized by distinct, invariable, and adequate meanings. In mathematics, each term is used in the same, invariable sense—does not this secure us against the fallacies of *fluctuating meanings?* No term is employed which is not a full and clear representative of the thing signified—are we not thus guarded against the illusions of obscurity? No expression involving a plurality of meanings is ever admitted—do we not thus become prepared to detect instinctively the sophistries of ambiguity? It hence appears that mathematics, in exacting an absolute strictness in their language, must conduce to an accuracy in the use of words and a skill in detecting their fallacies, which enable us instead of groping in obscurity in the field of probability to advance steadily amidst the obstructions that surround us, with the greatest assurance of a rapid and enlightened progress.

IV.

Their Value as an Instrument of Material Progress.

The time at my disposal will permit little more than a reference to the objective utility of mathematics as shown in their necessary connection with other sciences, and with the progress of mankind. It may be said that it is illogical to attempt to enhance the value of mathematics as a means of education by an appeal to their value as essential to human progress. But, in determining "what knowledge is of most worth" in education, it is not only proper but necessary to take into account its influence on material progress. For what is this progress, but the conquest of human liberty and intelligence over matter and material phenomena? It is certainly a part of the destiny of man to achieve such a conquest. In the earlier stages of civilization, but few of the secrets of nature are given up to man, and material forms contribute but little to his happiness—he does not yet appear as the master of his habitation. But soon the world begins to change its aspect before the operations of intelligence—it surrenders its secrets and its treasues and acknowledges its subjection to its appointed Master. Material progress is therefore but the reflex of intellectual development. Now, it must be admitted that the most effectual triumphs of mind over matter, have been won through mathematics. Take away from what has been secured to civilization through the long

(CONTINUED PAGE 20.)